1.a) $\min W = 240y_1 + 700y_2 + 20y_4$

$$\text{s.t.:}\begin{cases} y_1 + 4y_2 + y_3 \geq 10 \\ y_1 + 2y_2 - y_3 \geq 20 \\ 4y_1 + y_2 - y_3 + y_4 \geq 40 \\ y_1, y_2 \geq 0 \\ y_3, y_4 \leq 0 \end{cases}$$

1.b) $x_A = 90;\ x_B = 70;\ x_C = 20$**.** In order to maximize the profit, 90 (x_A) units of **A**, 70 (x_B) of **B** and 20 (x_C) of **C,** should be produced every week.

1.c) $y_1 = 15;\ x_1 = |240 - 240| = 0$. The h.m. available per week are fully used, so it is a scarce resource ($x_1 = 0$). For each additional h.m. (less) the weekly profit increases (decreases) 15 m.u. ($= y_1$), while optimal basis is kept.

$y_2 = 0;\ x_2 = |700 - 520| = 180$. Each week, there is a leftover of 180 m^3 ($= x_2$) in the oven capacity. So its internal value is null ($y_2 = 0$). Changes in its values does not cause changes in the weekly profit, while optimal basis is kept.

$y_3 = -5;\ x_3 = |0 - 0| = 0$. The relation imposed among the sales of products is satisfied at equality ($x_3 = 0$). For each additional unit (less) in this constraint the weekly profit decreases (increases) 5 m.u. ($= -y_1$), while optimal basis is kept.

$y_4 = -25;\ x_4 = |20 - 20| = 0$. The production of C is the minimum required ($x_4 = 0$). For more (less) each unit required the weekly profit decreases (increases) 25 m.u. ($= -y_4$), while optimal basis is kept.

1.d.1) $\text{Max } Z = 10x_A + 20x_B + 40x_C$

$$\text{s.t.:}\begin{cases} x_A + x_B + 4x_C + x_1 = 240 \\ 4x_A + 2x_B + x_C + x_2 = 700 \\ x_A - x_B - x_C - x_3 = 0 \\ x_A, x_B, x_C, x_1, x_2, x_3 \geq 0 \end{cases}$$

1.d.2) The last constraint should be multiplied by (-1). CE: $\text{Min}\{-10;\ -20;\ -40\} = -40 \ \rightarrow \mathrm{x_C}$;

CS: $\text{Min}\left\{\frac{240}{4};\ \frac{700}{1};\ \frac{0}{1}\right\} = 0 \ \rightarrow \mathrm{x_3}$

	Z	x_A	x_B	x_C	x_1	x_2	x_3	RHS
Z	1	-10	-20	-40	0	0	0	0
x_1	0	1	1	4	1	0	0	240
x_2	0	4	2	1	0	1	0	700
x_3	0	-1	1	1	0	0	1	0
Z	1	-50	20	0	0	0	40	0
x_1	0	5	-3	0	1	0	-4	240
x_2	0	5	1	0	0	1	-1	700
x_C	0	-1	1	1	0	0	1	0

$\mathbf{x} = (0, 0, 0, 240, 700, 0)$. Non optimal BFS. The OF row still has negative values and every solution in a simplex tableaux is basic.

1.d.3) BV: $x_1,\ x_2, x_C$. NBV: $x_A,\ x_B,\ x_3$.

1.d.4) The optimal value can never be smaller than the initial as the problem is a maximization problem and the FR contains the FR of the initial problem.